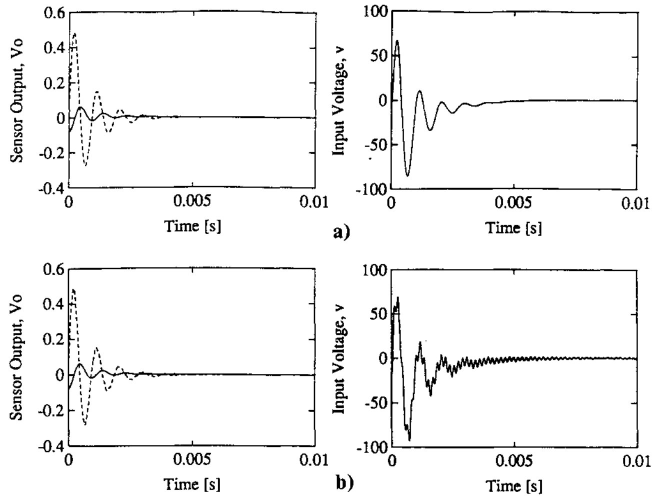

Fig. 3 Closed-loop response to nonzero initial deflection ($P = 3.8P_{cr,1}$): a) reduced order model and b) extended evaluation model.

controlled modes are reconstructed from the output of the extended model using a comb filter selecting modes 1 and 5 (an observer serves this purpose) and these estimates are used for feedback.

The resulting closed-loop response of the augmented system to nonzero initial conditions and the required input voltage to the actuators are shown in Fig. 3b. From Figs. 3a and 3b it can be seen, as expected, that there is no significant effect of the uncontrolled modes on the dynamics of the controlled modes.

Conclusion

In this Note we addressed the problem of buckling control using smart materials, a static instability of axially loaded members of a structure. We showed that the buckling of a flexible beam can be postponed beyond the first critical load by means of feedback using piezoelectric actuators and strain gauge sensors. It is observed that a controller design based on a fixed axial load P_{max} stabilizes the modeled modes for any $P \leq P_{max}$ and, therefore, is robust to slow load variations. Hence buckling in the first mode is inhibited, and the beam can support a load up to the second critical load. Actuator and sensor placement is discussed with regard to problems of spillover. Finally, spillover has not posed serious problems as we are able to design the controller, in the case of a beam, using a low-order model and verify stability for a high-order model.

Acknowledgment

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Robust Time-Optimal Control of Uncertain Structural Dynamic Systems

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Introduction

THE problem of computing robust time-optimal control inputs for uncertain flexible spacecraft has been recently investigated in Refs. 1 and 2. The primary control objective in such a robust control problem is to achieve a fast maneuvering time with minimum structural vibrations during and/or after a maneuver in the face of modeling uncertainty. In this Note we treat a similar problem to explore the effects of using a pair of noncollocated, one-sided jets. A parameter optimization approach, with additional constraints for performance robustness with respect to modeling uncertainty, is employed to solve such an interesting control problem.

A simple mathematical model of an uncertain structural dynamic system with one rigid-body mode and two flexible modes, as shown in Fig. 1, is used to illustrate the concept and methodology. Such a simple model is often used to represent an actual spacecraft with few dominant structural modes for the purposes of practical control design.[3,4] We consider a special case in which the structural flexibility and mass distribution of the system are quite uncertain, although the total mass (or inertia) of the system is well known. Consequently, we focus on the robust control problem of flexible structures in the face of modal frequency uncertainty as well as mode shape uncertainty. However, many theoretical issues inherent to constrained parameter optimization problems and the practical implementation issue inherent to any open-loop control approach are not discussed in this Note.

Problem Formulation

Consider the mass-spring model shown in Fig. 1, which is, in fact, a generic representation of a spacecraft with one rigid-body mode and two flexible modes. The modal equations of this system can be represented as

$$\ddot{y}_1 = \phi_{11}u_1 + \phi_{12}u_2 + \phi_{13}u_3 \tag{1a}$$

$$\ddot{y}_2 + \omega_2^2 y_2 = \phi_{21}u_1 + \phi_{22}u_2 + \phi_{23}u_3 \tag{1b}$$

$$\ddot{y}_3 + \omega_3^2 y_3 = \phi_{31}u_1 + \phi_{32}u_2 + \phi_{33}u_3 \tag{1c}$$

where y_i is the ith modal coordinate, ω_i the ith modal frequency, ϕ_{ij} the modal input distribution coefficients, and u_i the control inputs. The nominal parameter values are assumed as $m_1 = m_2 = m_3 = k_1 = k_2 = 1$ with appropriate units, and time is in units of second.

Using this simple generic model, we explore three cases: 1) case 1 with both "positive" and "negative" jets placed at body 1, 2) case 2 with a positive jet at body 1 and a negative jet at body 2, and 3) case 3 with a positive jet at body 1 and a negative jet at body 3. Case 1 is a typical case in which two opposing jets are collocated. In cases 2 and 3, two opposing jets are not collocated. In this Note, we consider only case 3, whereas detailed results for cases 1 and 2 can be found in Ref. 5.

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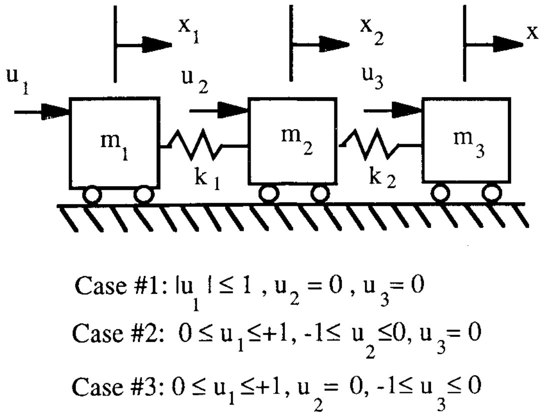

Fig. 1 Uncertain dynamical system with one rigid-body mode and two flexible modes.

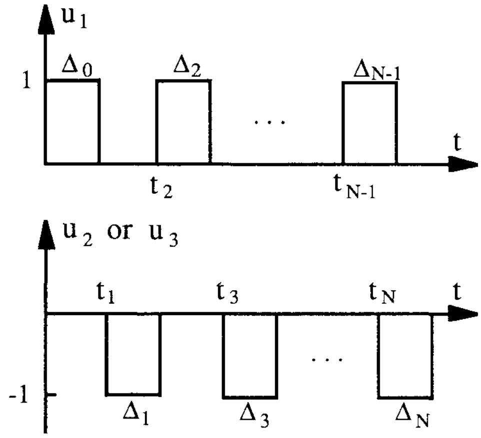

Fig. 2 Pulse sequences.

Time-Optimal Control (Case 3)

The modal equations for case 3 with the nominal parameter values become

$$\ddot{y}_1 = 0.3333(u_1 + u_3) \tag{2a}$$

$$\ddot{y}_2 + \omega_2^2 y_2 = 0.5(u_1 - u_3) \tag{2b}$$

$$\ddot{y}_3 + \omega_3^2 y_3 = 0.1667(u_1 + u_3) \tag{2c}$$

where $\omega_2 = 1$ rad/s, $\omega_3 = \sqrt{3}$ rad/s, and the two one-sided control inputs are bounded as

$$0 \le u_1 \le +1 \tag{3a}$$

$$-1 \le u_3 \le 0 \tag{3b}$$

The time-optimal control inputs are assumed to have two pulses per input, defined as in Fig. 2. The rest-to-rest maneuver constraints with $y_1(t_f) = 1$ can be found as[5]:

$$\Delta_0 - \Delta_1 + \Delta_2 - \Delta_3 = 0$$

$$6 + \sum_{j=0}^{3} (-1)^j [\Delta_j^2 + 2t_j \Delta_j^2] = 0$$

$$\sum_{j=0}^{3} (-1)^j \left\{ \sin(\omega_i t_j) - \sin\left[\omega_i (t_j + \Delta_j)\right] \right\} = 0, \qquad i = 2,3$$

$$\sum_{j=0}^{3} (-1)^j \left\{ \cos(\omega_i t_j) - \cos\left[\omega_i (t_j + \Delta_j)\right] \right\} = 0, \qquad i = 2,3$$

$$t_1, t_2, t_3, t_4, t_5 > 0; \qquad t_0 = 0$$

$$\Delta_j \ge 0$$

The time-optimal control solution can then be obtained by solving the constrained minimization problem

$$\min J = t_f = t_3 + \Delta_3 \tag{4}$$

subject to the constraints given earlier. A solution to this problem can be obtained using a standard IMSL subroutine as

$$\begin{aligned} t_0 &= 0.0000, & \Delta_0 &= 0.9510 \\ t_1 &= 1.3631, & \Delta_1 &= 0.1658 \\ t_2 &= 2.8329, & \Delta_2 &= 0.1658 \\ t_3 &= 3.4109, & \Delta_3 &= 0.9510 \\ & & t_f &= 4.3619 \end{aligned} \tag{5}$$

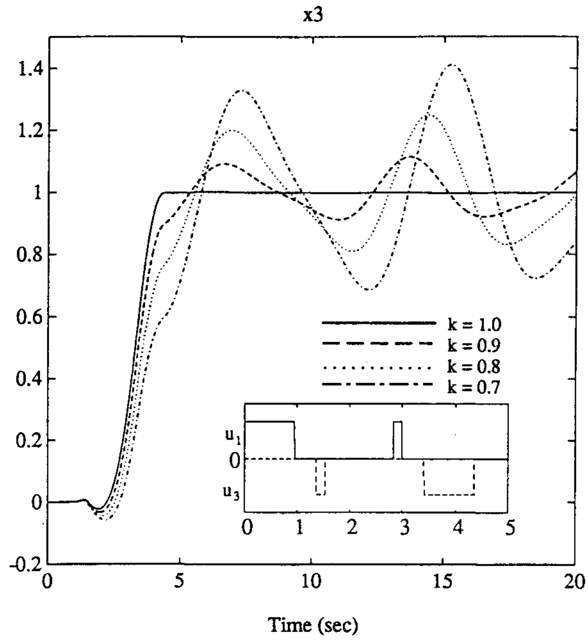

Fig. 3 Responses to time-optimal control input (case 3).

The time responses of x_3 to the time-optimal control inputs are shown in Fig. 3 for four different values of $k = k_1 = k_2$. It can be seen that the responses are quite sensitive to variations in the model parameters. However, an interesting feature of this case is that the pulse sequences are of a "bang-off-bang" type, resulting in the control on time of 2.234 s, which is different from the maneuver time of 4.362 s. Compared to cases 1 and 2, this case has the fastest maneuver time as well as the smallest control on time.[5] Therefore, the actuator configuration of case 3 may be considered to be "optimal" in the sense of minimizing both the maneuver time and the jet on time.

Robust Time-Optimal Control (Case 3)

As shown in the preceding section, a standard, time-optimal control approach requires an accurate mathematical model, and thus the resulting solution is often sensitive to plant modeling uncertainty. In this section, a parameter optimization problem is formulated with additional constraints for robustness with respect to the structural frequency uncertainty: $\mathrm{d}y_i(t)/\mathrm{d}\omega_i = 0$ for $t \ge t_f$. The resulting *robustified* or *desensitized* time-optimal solution is a multiswitch bang-bang control and is thus implementable for spacecraft equipped with on-off reaction jets.[3,4]

The robustness constraints for case 3 with two one-sided control inputs can be found as[5]:

$$-\phi_{i1}\sum_{j=0,2}^{N-1} c_{ij} + \phi_{i3}\sum_{j=1,3}^{N} c_{ij} = 0 \tag{6a}$$

$$\phi_{i1}\sum_{j=0,2}^{N-1} s_{ij} - \phi_{i3}\sum_{j=1,3}^{N} s_{ij} = 0 \tag{6b}$$

where

$$c_{ij} = t_j\cos(\omega_i t_j) - (t_j+\Delta_j)\cos\left[\omega_i(t_j+\Delta_j)\right]$$

$$s_{ij} = t_j\sin(\omega_i t_j) - (t_j+\Delta_j)\sin\left[\omega_i(t_j+\Delta_j)\right]$$

Assuming that each control input has three pulses, we have 11 unknowns to be determined, the t_j and Δ_j defined as in Fig. 2. The robust time-optimal solution can then be obtained by solving the constrained parameter optimization problem

$$\min J = t_5 + \Delta_5 \tag{7}$$

subject to

$$\Delta_0 - \Delta_1 + \Delta_2 - \Delta_3 + \Delta_4 - \Delta_5 = 0$$

$$6 + \sum_{j=0}^{5}(-1)^j[\Delta_j^2 + 2t_j\Delta_j^2] = 0$$

$$\sum_{j=0}^{5}(-1)^j\left\{\sin(\omega_i t_j) - \sin\left[\omega_i(t_j+\Delta_j)\right]\right\} = 0, \qquad i = 2,3$$

$$\sum_{j=0}^{5}(-1)^j\left\{\cos(\omega_i t_j) - \cos\left[\omega_i(t_j+\Delta_j)\right]\right\} = 0, \qquad i = 2,3$$

$$\sum_{j=0}^{5}(-1)^j\left[t_j\cos\omega_i t_j - (t_j+\Delta_j)\cos\omega_i(t_j+\Delta_j)\right] = 0, \qquad i = 2,3$$

$$\sum_{j=0}^{5}(-1)^j\left[t_j\sin\omega_i t_j - (t_j+\Delta_j)\sin\omega_i(t_j+\Delta_j)\right] = 0, \qquad i = 2,3$$

$$\Delta_j \geq 0; \qquad j = 0, 1, 2, 3, 4, 5$$

$$t_1, t_2, t_3, t_4, t_5 > 0; \qquad t_0 = 0$$

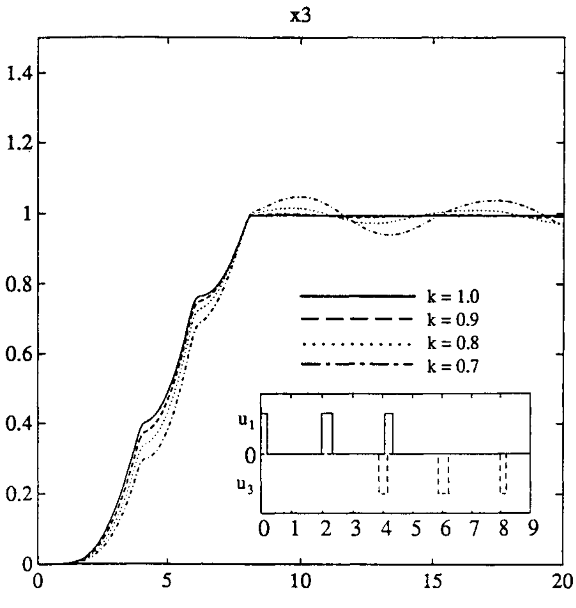

Fig. 4 Responses to robust time-optimal control input (case 3).

Table 1 Summary of the results

	$J^* = t_f$, s	Jet on time
	Time-optimal control	
Case 1	6.511	6.511
Case 2	5.340	4.172
Case 3	4.362	2.234
	Robust time-optimal control	
Case 1	10.18	10.18
Case 2	7.360	4.765
Case 3	8.187	1.678

A solution to this problem is obtained as

$$\begin{aligned} t_0 &= 0.000, & \Delta_0 &= 0.2189 \\ t_1 &= 3.774, & \Delta_1 &= 0.2609 \\ t_2 &= 2.030, & \Delta_2 &= 0.3594 \\ t_3 &= 5.778, & \Delta_3 &= 0.3594 \\ t_4 &= 4.152, & \Delta_4 &= 0.2609 \\ t_5 &= 7.968, & \Delta_5 &= 0.2189 \end{aligned} \tag{8}$$

$$t_f = 8.187$$

The time responses of x_3 to the robust time-optimal control inputs are shown in Fig. 4 for four different values of $k = k_1 = k_2$. It can be seen that the robustness has been increased at the expense of the increased maneuvering time of 8.187 s, as compared to the ideal minimum time of 4.362 s. However, the control on time is only 1.678 s! Because of the properly coordinated pulse sequences, the flexible modes are not significantly excited during maneuvers and the residual responses after the maneuvers are well desensitized.

Table 1 summarizes the results for all three cases. As can be noted in this table, it is natural to select the actuator configuration of case 3, since this case provides the "best" overall performance in the sense of minimizing the maneuvering time, fuel consumption (jet on time), and structural mode excitation. For case 1, the maneuvering time and the jet on time are the same, which is clearly undesirable from the viewpoint of fuel consumption.

Conclusions

A time-optimal open-loop control problem of flexible spacecraft in the face of modeling uncertainty has been investigated. The results indicate that the proposed approach significantly reduces the residual structural vibrations caused by modeling uncertainty. The results also indicate the importance of a proper jet placement for practical tradeoffs among the maneuvering time, fuel consumption, and performance robustness. It is again emphasized that simply prolonging the maneuver time does not help to reduce residual structural vibrations caused by modeling uncertainty; a proper coordination of pulse sequences is necessary. However, a further research is needed for designing a closed-loop, on-off controller with control inputs similar to those of a robust time-optimal (open-loop) controller.

Acknowledgments

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Control and Filtering of Wide-Band Noise Driven Linear Systems

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Introduction

THE modern stochastic optimal control and filtering theories use white noise driven systems. The results such as Kalman filtering (originally described in Kalman[1]) and separation principle (see Wonham[2]) are based on the white noise model. Indeed, a white noise, being a mathematical idealization, gives only an approximate description to the real noise processes. In some fields the parameters of the real noises are near to the parameters of the white noise (for example, in telemetry[3]) and so the mathematical methods of optimal control and filtering for the white noise driven systems can be satisfactorily applied in them. But, in many fields the white noise is a crude approximation of the real noises. Therefore, adequate modeling of the processes in such fields must use a wide-band noise model of the real noises.

Many authors investigate the optimal control and filtering problems for the wide-band noise driven systems using robustness or approximate approaches (see, for example, Kushner and Runggaldier[4]). A different approach is suggested in Bashirov[5] which is based on the representation of the wide-band noise as a distributed delay of the white noise. Such an approach gives the possibility of extending the fundamental separation principle and Kalman filtering ideas to the wide-band noise driven linear systems and of obtaining direct formulas for optimal control and filter for such systems.[5,6] In this Note we present these results in a simple case. The aim of this Note is to introduce the specialists of aerospace engineering to the new optimal control and filtering results for the wide-band noise driven systems from Bashirov[5,6] that can be successfully applied in various engineering practice. The related paper on application of wide-band noise Kalman filtering in gravimetry is Bashirov et al.[7]

Wide-Band Noise

As mentioned the white noise is only an approximate model of the real noises. Thus if we use the optimal control and filtering results for white noise driven systems in application to the real systems then our outputs are not optimal and, indeed, might be quite far from being optimal.

The issue is that in reality the noises are marked with a property which ensures correlation of their values within a small time interval, i.e., if we denote such noise by φ, then

$$\operatorname{cov}\,[\varphi(t+s),\ \varphi(t)] = \begin{cases} \Lambda(s), & 0 \le s < \epsilon \\ 0, & s \ge \epsilon \end{cases} \qquad t \ge 0 \tag{1}$$

where $\epsilon > 0$ is a small value. The random process φ with the property in Eq. (1) is called a stationary wide-band noise.[8] If ϵ is so small in Eq. (1) that it is normally assumed to be 0, then the wide-band noise is transformed into a white noise. As noted earlier, in many fields such a substitution of a wide-band noise by a white noise gives rise to tangible distortions. Therefore, mathematical methods of optimal control and filtering for wide-band noise driven systems need to be developed. We present such a method from Bashirov[5,6] which generalizes the Kalman filtering and separation principle.

The underlying idea of our method consists in representation of a wide-band noise φ in the form of the distributed delay of a white noise, i.e.,

$$\varphi(t) = \int_{-\epsilon}^{0} \Phi(\theta) w(t+\theta)\, \mathrm{d}\theta, \qquad t \ge 0 \tag{2}$$

where w is a vector white noise, $\epsilon = \text{const} > 0$, and Φ is a deterministic matrix function. If $\operatorname{cov}[w(t),\ w(s)] = W\delta(t-s)$ where δ is a delta function, then $\operatorname{cov}[\varphi(t+s),\ \varphi(t)] = 0$ with $s \ge \epsilon$ and

$$\operatorname{cov}[\varphi(t+s),\ \varphi(t)] = \int_{s-\epsilon}^{0} \Phi(\theta - s) W \Phi^*(\theta)\, \mathrm{d}\theta \tag{3}$$

with $0 \le s < \epsilon$, i.e., φ is a stationary wide-band noise.

Presence of a wide-band noise, Eq. (2), in real systems can be formally explained by vibration; for example, at the moment t the vibration that is formed by the action of the white noise w during the time between $t-\epsilon$ and t affects the system. The values of the white noise w until the moment $t-\epsilon$ do not take part in the formation of the vibration at the moment t, because at the moment t the vibration from them is small enough that we can neglect it in the mathematical model (2). Consequently, the parameters Φ and ϵ of the wide-band noise in Eq. (2) have the following meaning: Φ stands for the coefficient of relaxing the initial effect w at different moments of time, and ϵ represents the interval within which the consequences of the disturbing noises are not present.

It should be noted that the wide-band noise described by Eq. (2) is very convenient for mathematical investigations of optimal control and filtering problems. Next we shall consider a somewhat different representation of a wide-band noise from Eq. (2), namely,

$$\varphi(t) = \int_{-\min(t,\epsilon)}^{0} \Phi(\theta) w(t+\theta)\, \mathrm{d}\theta, \qquad t \ge 0 \tag{4}$$

which differs from Eq. (2) only when $0 \le t < \epsilon$. The wide-band noise in Eq. (4) corresponds to the real case when vibration by the white noise w starts from the initial moment 0 and, therefore, when $0 \le t < \epsilon$ the wide-band noise φ is formed by the values $w(\theta)$, $0 \le \theta \le t$.

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